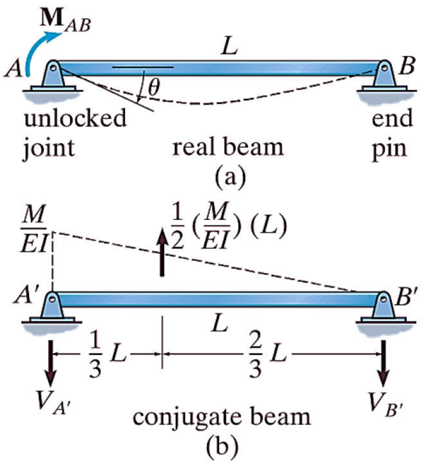
9.2 Stiffness Factor Modification:

In the previous examples of moment distribution we have considered each beam span to be constrained by fixed support at its far end when distributing and carrying over the moments. It's possible to modify the stiffness factor as the following:

9.2.1 Member Pin supported at far end



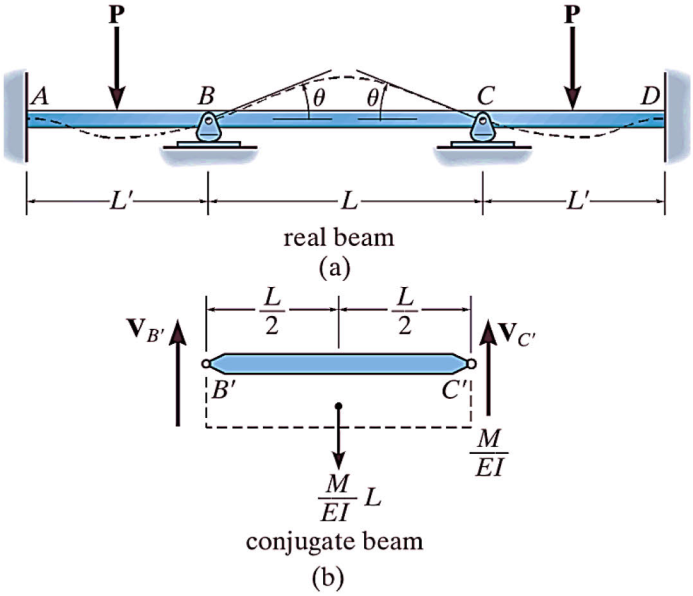
$$\sum M_{B'} = 0 \Rightarrow V_A'(L) - \frac{M}{2EI}\cdot L \cdot \frac{2}{3}L$$

$$V_A' = \theta = \frac{ML}{3EI}, \qquad M = \frac{3EI\theta}{L}$$

$$K = \frac{3EI}{L}$$ Far End Pinned or Roller Supported

9.2.2 Symmetric Beam and Loading.

If a beam is symmetric with respect to both its loading and geometry, the bending-moment diagram for the beam will also be symmetric. As a result, a modification of the stiffness factor for the center span can be made, so that moments in the beam only have to be distributed through joints lying on either half of the beam.



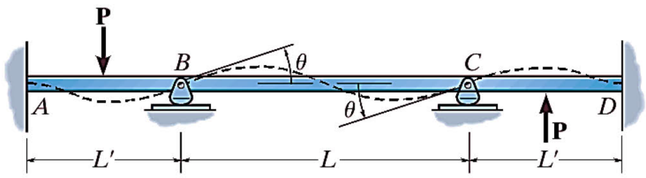
$$\sum M_{C'} = 0 \Rightarrow -V_B'(L) + \frac{M}{EI}\cdot L \cdot \frac{L}{2}$$

$$V_A' = \theta = \frac{ML}{2EI}, \quad \Rightarrow M = \frac{2EI\theta}{L}$$

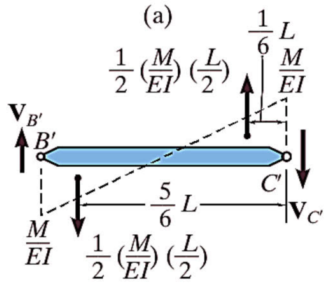
$$K = \frac{2EI}{L}$$, Symmetric Beam and Loading

9.2.3 Symmetric Beam with Antisymmetric Loading.

If a symmetric beam is subjected to antisymmetric loading, the resulting moment diagram will be antisymmetric.



real beam
(a)



conjugate beam
(b)

$\sum M_{C'} = 0;$

$$-V_{B'}(L) + \frac{1}{2}\left(\frac{M}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{5L}{6}\right) - \frac{1}{2}\left(\frac{M}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = 0$$

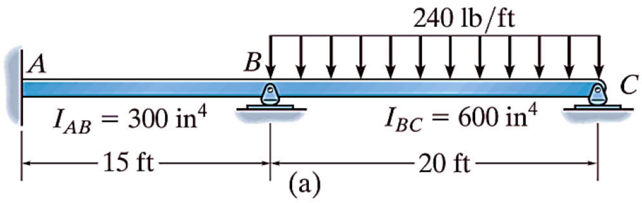
$$V_{B'} = \theta = \frac{ML}{6EI}$$

$$M = \frac{6EI}{L}\theta$$

$K = \frac{6EI}{L}$ Symmetric Beam with Antisymmetric Loading

EXAMPLE 9.2.1

Use moment distribution method to determine the moment at joint ***A***, ***B***, and ***C***, for the beam shown in **Fig. *a***. ***EI*** is constant.



(a)

Solution

Stiffness Factor:

$$K_{AB} = \frac{4EI}{L} = \frac{4(E)300}{15} = 80E \ , \quad K_{BC} = \frac{3EI}{L} = \frac{3(E)600}{20} = 90E$$

$$K_{AB} : K_{BC} = 80 : 90$$

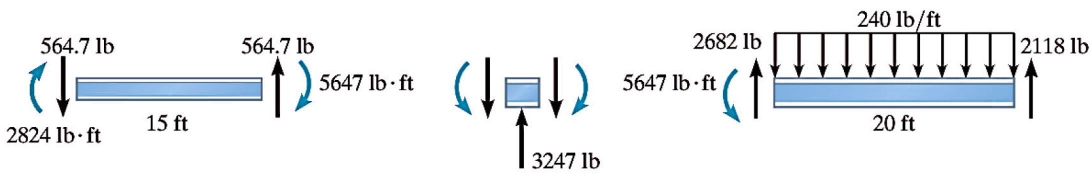
Distribution Factor:

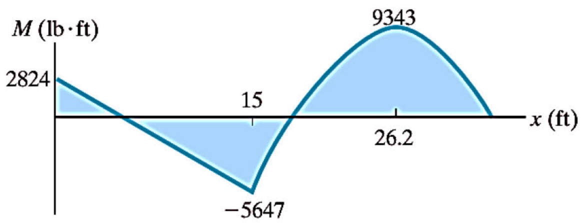
$$DF_{AB} = \frac{80}{170} = 0.4706 \ , \quad DF_{BC} = \frac{90}{170} = 0.5294$$

Fixed-End Moments (FEMs):

$$(FEM)_{BC} = -\frac{wL^2}{8} = -\frac{240(20)^2}{8} = -12000 \text{ lb.ft}$$

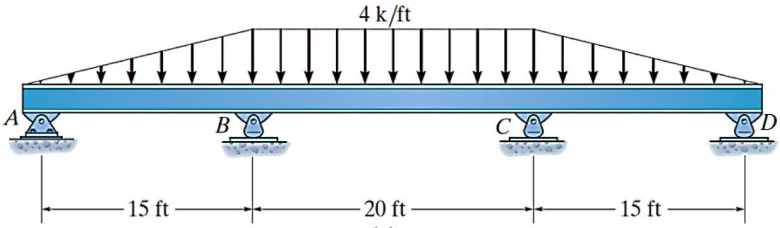
Joint	*A*	*B*		*C*
Member	*AB*	*BA*	*BC*	*CB*
DF	0	0.4706	0.5294	1
FEM			-12000	
Dist. CO.		5647.2	6352.8	
Dist. CO.	2823.6			0
∑*M*	2823.6	5647.2	-5647.2	0





EXAMPLE 9.2.2

Determine the internal moments at the supports for the beam shown in **Fig. *a***. ***EI*** is constant**.**



(a)

Solution

The beam and loading and loading are symmetrical, we will apply ***K* = 2*EI*/*L*** to compute the stiffness factor of the center span ***BC*** and therefore use only the ***left half*** of the beam for the analysis. Furthermore, the distribution of moment at ***A*** can be skipped by using the ***FEM*** for a triangular loading on a span with one end fixed and the other pinned.

Member *AB* &*BC*

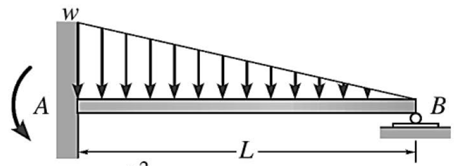
Stiffness Factor:

$$K_{BA} = \frac{3EI}{15} = \frac{EI}{5}\ , \qquad K_{BC} = \frac{2EI}{20} = \frac{EI}{10}$$

$$K_{BA} : K_{BC} = 10:5 = 2:1$$

Distribution Factor:

$$DF_{BA} = \frac{2}{3} = 0.667\ , \qquad DF_{BC} = \frac{1}{3} = 0.334$$



$(FEM)'_{AB} = \frac{wL^2}{15}$

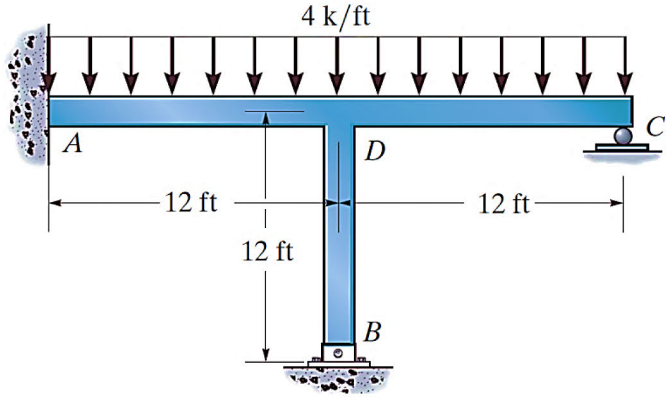
Fixed-End Moments (FEMs):

$$(FEM)'_{BA} = \frac{wL^2}{15} = \frac{4(15)^2}{15} = 60 \text{ k.ft}\ , \quad (FEM)_{BC} = -\frac{wL^2}{12} = -\frac{4(20)^2}{12} = -133.3 \text{ k.ft}$$

Joint	*A*	*B*	
Member	***AB***	***BA***	***BC***
DF	**1**	**0.667**	**0.333**
FEM		**60**	**-133.3**
Dist. CO.		**48.9**	**24.4**
∑*M*	**0**	**108.9**	**-108.9**

EXAMPLE 9.2.3

Determine the moments at the fixed support *A* and joint *D*. Assume *B* is pinned.



Solution

Member Stiffness Factor

$$K_{AD}=\frac{4EI}{12}=\frac{EI}{3}\ ,\ K_{DC}=K_{DB}=\frac{3EI}{12}=\frac{EI}{4}$$

Distribution Factor:

$$DF_{AD}=0$$

$$DF_{DA}=\frac{EI/3}{EI/3+EI/4+EI/4}=0.4\ ,\ DF_{DC}=DF_{DB}=\frac{EI/4}{EI/3+EI/4+EI/4}=0.3$$

$$DF_{CD}=DF_{BD}=1$$

Fixed-End Moments (FEMs):

$$(FEM)_{AD}=-\frac{wL^2}{12}=-\frac{4(12)^2}{12}=-48\text{ k.ft}\qquad (FEM)_{DA}=\frac{wL^2}{12}=\frac{4(12)^2}{12}=48\text{ k.ft}$$

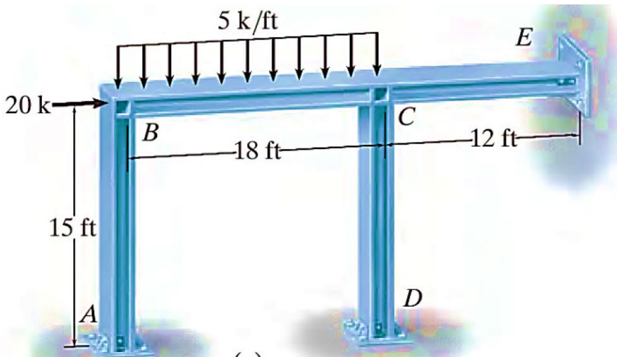
$$(FEM)_{DC}=-\frac{wL^2}{8}=-\frac{4(12)^2}{8}=-72\text{ k.ft}$$

$$(FEM)_{CD}=(FEM)_{BD}=(FEM)_{DB}=0$$

Joint	*A*	*D*			*C*	*B*
Member	***AD***	***DA***	***DB***	***DC***	***CD***	***BD***
DF	**0**	**0.4**	**0.3**	**0.3**	**1**	**1**
FEM	**-48**	**48**	**0**	**-72**	**0**	**0**
Dist. CO.		**9.60**	**7.20**	**7.20**		
Dist. CO.	**4.80**					
$\sum M$	**-43.2**	**57.6**	**7.20**	**-64.8**	**0**	**0**

EXAMPLE 9.2.4

Determine the internal moments at the joints of the frame shown in **Fig. *a***. There is a pin at ***E*** and ***D*** and a fixed support at ***A***. ***EI*** is constant.



(a)

Solution

Member *AB* &*BC*

Stiffness Factor:

$$K_{BA}=\frac{4EI}{15}\ ,\ K_{BC}=\frac{4EI}{18}$$

$$K_{BA}:K_{BC}=18:15=6:5$$

Distribution Factor:

$$DF_{BA}=\frac{6}{11}=0.545\ ,\quad DF_{BC}=\frac{5}{11}=0.455\ ,\quad DF_{AB}=0$$

Member *CB* , *CD* & *CE*

Stiffness Factor:

$$K_{CB}=\frac{4EI}{18}=0.222EI\ ,\ K_{CD}=\frac{3EI}{15}=0.2EI\ ,\ K_{CE}=\frac{3EI}{12}=0.25EI$$

$$K_{CB}:K_{CD}:K_{CE}=0.222:0.2:0.25$$

Distribution Factor:

$$DF_{CB}=\frac{0.222}{0.222+0.2+0.25}=0.33$$

$$DF_{CD}=\frac{0.2}{0.222+0.2+0.25}=0.298\qquad DF_{DC}=1\ ,\ DF_{EC}=1$$

$$DF_{CD}=\frac{0.25}{0.222+0.2+0.25}=0.372$$

Fixed-End Moments (FEMs):

$$(FEM)_{BC}=-\frac{wL^2}{12}=-\frac{5(18)^2}{12}=-135\text{ k.ft}\ ,\qquad (FEM)_{CB}=\frac{wL^2}{12}=\frac{5(18)^2}{12}=135\text{ k.ft}$$

Joint	*A*	*B*		*C*			*D*	*E*
Member	***AB***	***BA***	***BC***	***CB***	***CD***	***CE***	***DC***	***EC***
DF	**0**	**0.545**	**0.455**	**0.330**	**0.298**	**0.372**	**1**	**1**
FEM			**-135**	**135**				
Dist. CO.		**73.6**	**61.4**	**- 44.6**	**-40.2**	**-50.2**		
FEM	**36.8**		**-22.3**	**30.7**				
Dist. CO.		**12.2**	**10.1**	**-10.1**	**-9.1**	**-11.5**		
FEM	**6.1**		**-5.1**	**5.1**				
Dist. CO.		**2.8**	**2.3**	**-1.7**	**-1.5**	**-1.9**		
FEM	**1.4**		**-0.8**	**1.2**				
Dist. CO.		**0.4**	**0.4**	**-0.4**	**-0.4**	**-0.4**		
FEM	**0.2**		**-0.2**	**0.2**				
Dist. CO.		**0.1**	**0.1**	**-0.1**	**0.0**	**-0.1**		
$\sum M$	**44.5**	**89.1**	**-89.1**	**115**	**-51.2**	**-64.1**		